Elastodynamic Contact Problems for Interface Cracks under Harmonic Loading

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2-D fracture dynamics’ problems for elastic bimaterials with cracks located at the bonding interface under the oblique time harmonic wave are considered in the study. The system of boundary integral equations for displacements and tractions is derived from Somigliana identity taking the contact interaction of the opposite crack faces into account. For the numerical solution the collocation method with piecewise constant approximation is used. The numerical results are obtained for various values of the angle of the wave incidence and the wave frequency taking the friction effects into account.

1 Introduction

The increasing number of applications in which composite materials are involved has caused a significant amount of publications devoted to fracture mechanics problems with interface cracks. The bonding interface is potentially critical region for crack formation, leading to weakening of mechanical strength and subsequent failure because of stress concentrations and rapid change of stress levels along the boundary. Since the cracks cannot be eliminated their behaviour must be predicted under different conditions and types of loading [1, 2]. The knowledge of fracture process and use of numerical models can help to avoid such usual causes of failure as improper material selection, wrong processing and improve the accuracy of the designs. The dynamic crack problems are usually solved on the assumption that the crack faces do not interact [2, 3]. However, the investigation of the contact iteration of the crack faces can play an important role in understanding of the dynamic interfacial crack problems. The interaction of opposite crack faces is a nonlinear process that depends strongly on the direction of externally applied load. The analysis of dynamic problems for elastic heterogeneous cracked solids with allowance for crack faces contact iteration may provide useful information for monitoring of the state of structures and materials [4, 5].

2 Methodology and problem description

Consider a linear crack located at the bimaterial interface between two isotropic and linear elastic half-spaces under action of the oblique harmonic wave. The stress strain state in both domains is defined by the linear Lamé equations for displacement field. It is assumed that the body is strainless at the initial moment and the continuity equations for displacement are satisfied at the bonding interface. In addition, the Sommerfeld radiation-type is satisfied at the infinity, i.e. the displacements at the bonding interface decrease gradually with increase in distance to the crack. When the material is subjected to the external dynamic loading, the mutual movement of opposite crack faces results in the contact interaction on the crack surface. The interaction of crack faces alters significantly the stress field near the crack front. The friction in the contact domain leads to the appearance of adherence and sliding of subdomains. The Signorini unilateral constraints are imposed for normal components of the displacements and contact forces, i.e. the contact force is unilateral and there is no interpenetration of the opposite crack faces. Moreover we assume that the friction on the contact surface is governed by the Coulomb friction law, which means that the opposite crack faces remain immovable with respect to each other in the tangential direction until they are held by the friction force, however, as soon as the magnitude of the contact forces reaches a certain limit, the slipping effect may occur. The components of the displacement and traction vectors are expanded into the Fourier exponential series. The appropriate Fourier coefficients are related to each other by the system of boundary integral equations, which has to be solved with taking the above mentioned constraints into account. The solution is refined during the iteration process, according to the iterative procedure given in [6, 7], until the distribution of physical values satisfy the contact constraints.

3 Numerical solution of the problem

As the numerical solution we contemplate a linear crack of the length $2a$ subject to the oblique harmonic loading of wave incidence of $30^\circ$ between steel and aluminium half-spaces with following properties: $E^{(1)} = 200GPa, E^{(2)} = 70GPa; \nu^{(1)} = 0.25, \nu^{(2)} = 0.34; \rho^{(1)} = 7800kg/m^3, \rho^{(2)} = 2700kg/m^3$. The distribution of displacements and tractions at the bonding interface and the crack’s surface are obtained and analyzed. The variation of normal and tangential components of

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contact forces on the entire crack surface during the oscillation cycle are obtained for different values of the angle of the wave incidence, $\alpha$; the wave number, $k_{2}^{(2)a}$; and the friction coefficient, $k_\tau$. The contact forces normalized by appropriate static magnitudes are presented in Fig. 1 for the friction coefficient $k_\tau = 0.2$ and the wave number $k_{2}^{(2)a} = 0.7$. The convergence of the algorithm as function of iteration number is given in Fig. 2. The influence of the loading frequency on the stress intensity factor distribution is investigated.

4 Conclusions

In the study, the normal and tangential components of the contact forces were computed for the case of the two-dimensional elastic heterogeneous solid with a linear interface crack under the oblique tension-compression harmonic wave. The problem was solved by the boundary integral equations method using an iterative algorithm, which brings displacements and forces in correspondence with the Signorini constraints and the Coulomb friction law. It enables to take into consideration not only the interaction of the opposite crack faces but also the effect of the friction.

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